Midterm #2 Review

April 22, 2026

Notes by Jenna May

General course objectives

- Intuition for signal processing concepts
- Quantify design tradeoffs in signal quality vs. runtime complexity

Signal Quality Measures

- SNR = signal power / noise power
  - Thermal noise modeled as Gaussian random process in rx analog rf front end
  - Quantization noise modeled as uniform in the quantizer in an A/D converter
- PAM/QAM bit rate: **J f_{sym}**
- PAM **bit rate ∝ Q($C_0\sqrt{SNR}$)**
- PAM error vector magnitude^2
  - Nearest Tx Constellation Point or Training Symbol

    $$(\hat{a}_n - a_n)^2$$

    Estimated Rx symbol amplitude
  - Objective function captures all the impairments in the error term $(\hat{a}_n - a_n)$
  - Adapt subsystem parameters (e.g. FIR channel equalizer coefficients) to minimize the objective function – works well during training where we know a_n

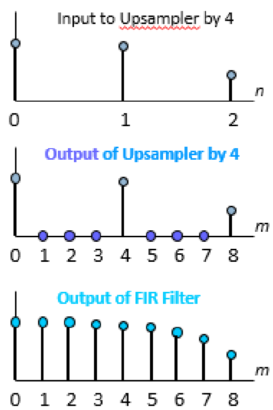
Building Blocks

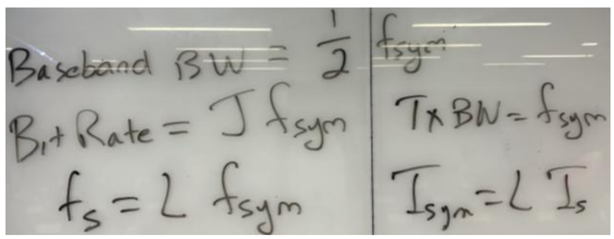
- Signals
  - Impulse
  - Sinusoids
  - Exponentials
  - Rectangular Pulse
  - Triangular Pulse
  - Sinc & Raised Cosine
  - Chirp
  - Pseudo-noise
  - Impulse train
  - Noise
- Systems
  - Adder & gain/multiplier
  - Ideal delay
  - FIR & IIR filters
  - Pointwise nonlinearities (*squarer, absolute value, etc.*)
  - Signal generation (*sinusoidal*)
  - Samplers & up/downsampling
  - Quantizers
  - Modulators/demodulators
  - Adaptation (*steepest descent*)
  - Fast Fourier transform

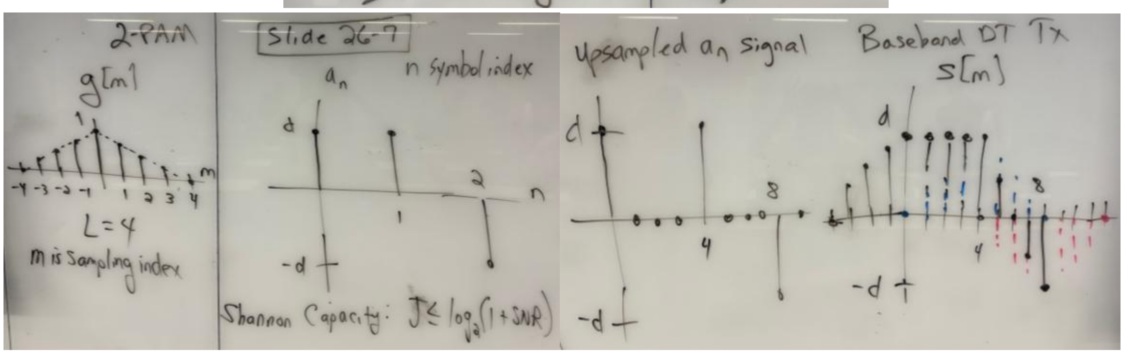
Increasing the Sampling Rate

- Interpolation: upsampling by L followed by an interpolation FIR filter g[m]
- Upsampler copies an input sample to the output and inserts L-1 zero values; repeat
  - Increases the sampling rate by a factor of L
  - Output contains L-1 high-frequency replicas of the input signal

- Interpolation FIR filter g[m]
  - LPF with **bandwidth π/L** to **remove high frequencies** introduced by upsampling
  - Having every Lth sample of g[m] be zero keeps the input values to the upsampler unchanged and fills in the inserted zero values with something reasonable
- In a baseband PAM/QAM transmitter
  - L is the number of samples in a symbol period
  - Upsampling by L increases the sampling rate from the symbol rate f_{sym} to the D/A sampling rate f_s.
  - The interpolation FIR filter g[m] is the pulse shaping filter to enforce the baseband bandwidth

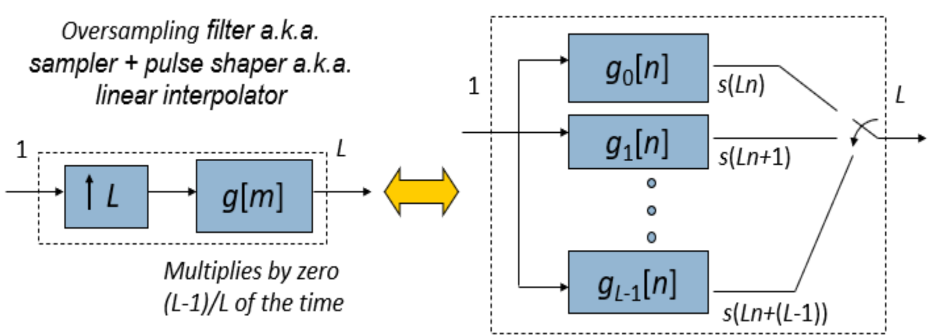






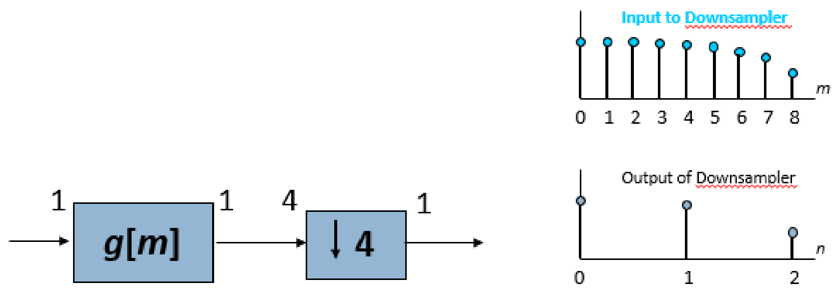
<u>Polyphase Filterbank</u>

- Avoids multiplications in the interpolation filter by the zeroes inserted by upsampling
  - **Saves factor of L** in multiplications and input storage
- **Split g[m] into L shorter polyphase filters** operating at the lower sampling rate

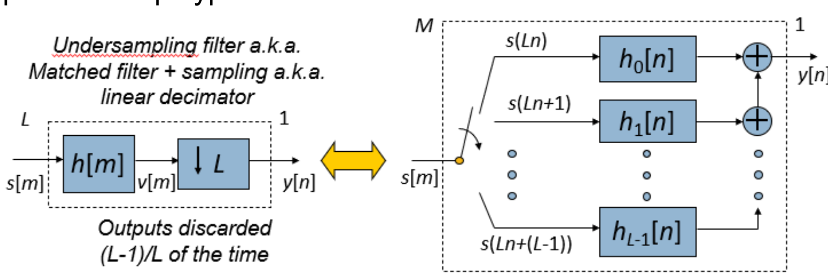


Decreasing sampling rate

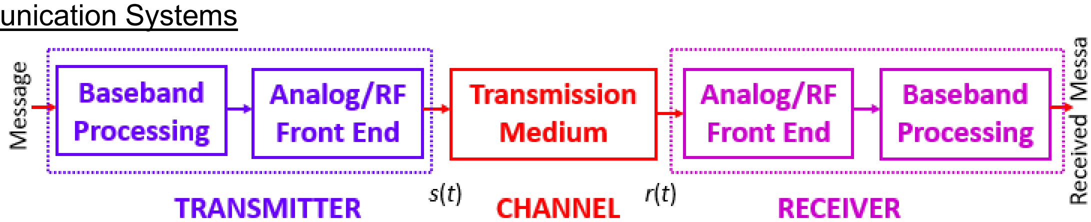
- Decimation: decimation FIR filter g[m] followed by downsampling by L
- Decimation FIR filter is an anti-aliasing lowpass filtering to enforce the Sampling Theorem due to downsampling
    - **Bandwidth: π/L**
- Downsampling **outputs first sample and discards L-1 input samples**
    - Decreases input sampling rate by factor of L on the output
- In a baseband PAM/QAM receiver,
    - L is the number of samples in a symbol period
    - Downsampling by L decreases the A/D sampling rate f_s to the symbol rate f_{sym}
    - Decimation FIR filter g[m] is the matched filter to enforce baseband bandwidth



- Can implement as polyphase filterbank



Communication Systems



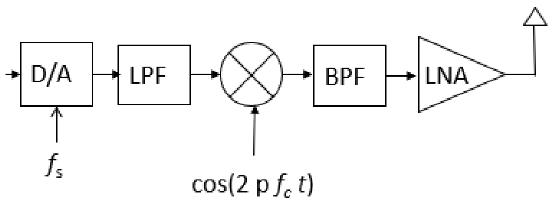
Transmitter

- Baseband message signal is information to be sent: voice, data, etc.
- Tx BB processing includes lowpass filtering to enforce transmission band
- Tx analog/RF front end includes **DAC, analog/RF upconverter, and transmit filter**

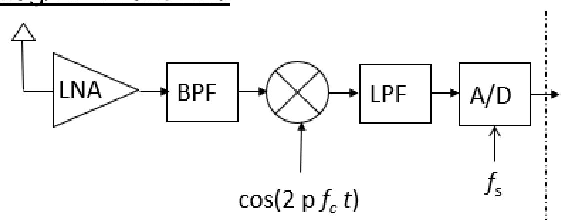
Receiver

- RF analog/RF front end includes **receive filter, carrier recovery, analog/RF downconverter, automatic gain control, ADC**.
- BB processing extracts and enhances BB signal

Tx Analog/RF

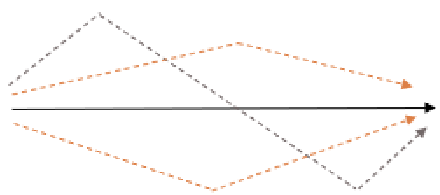


Rx Analog/RF Front End

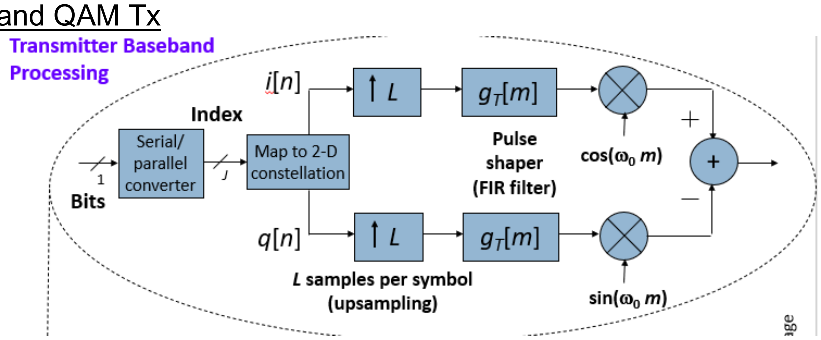


Channel Modeling

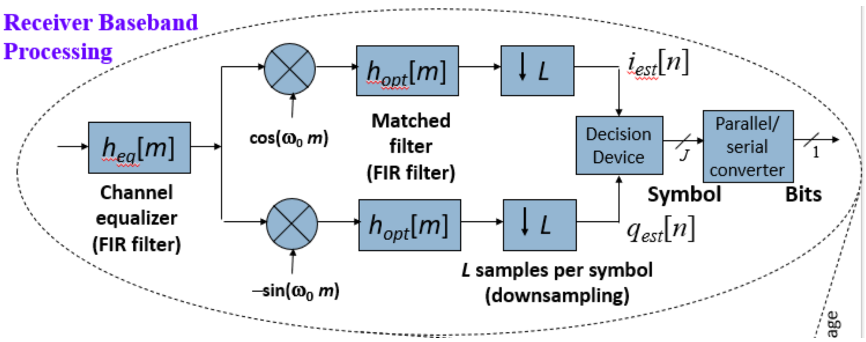
- Electromagnetic propagation of transmitted waves to the receiver
- Each path from tx to rx will have a different attenuation and delay
  - Distance traveled" d = c * delay where c is the speed of light [m/s]
  - Delay: d / c
  - Attenuation: d^{exp} where exp is between -3 and -4. In a vacuum, exp = -2.



Baseband QAM Tx



Baseband QAM Rx



QAM Signal Quality

- Assumptions
  - Each symbol is equally likely
  - Channel only contains AWGN, noise power $2\sigma^2$
  - Carrier frequency and phase recovery
  - Symbol timing recovery
- **Probability of symbol error** for 16 QAM

$$P(e) = 3Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right) - \frac{9}{4}Q^2\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$$

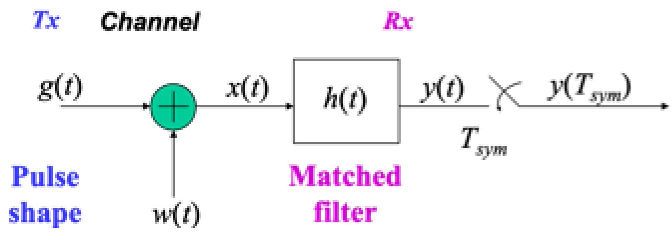
$$\frac{d}{\sigma}\sqrt{T_{sym}} \text{ is proportional to } \sqrt{\text{SNR}}$$

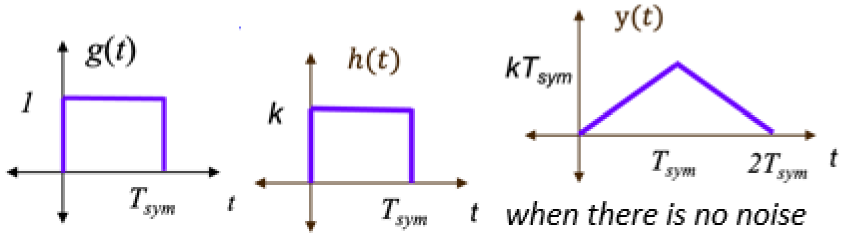
Communication System Tradeoffs

- How are parameters affected if only parameter in first columns **increases?**
  - Blank means no effect
- Tip to check understanding: explain justification behind each answer

Parameter	Transmission Bandwidth	Bit Rate	Symbol Error Rate	Tx Power Consumption	Run-Time Complexity
B bits in A/D output & D/A input		?	?		increase
2d constellation spacing in Volts			decrease	increase	
f_{sym} symbol rate in Hz	increase	increase	increase		increase
J bits/symbol		increase	increase	increase	increase
L samples/symbol			decrease		increase
N_g symbol periods in pulse shape			decrease		increase

<u>Symbol Timing Recovery</u>

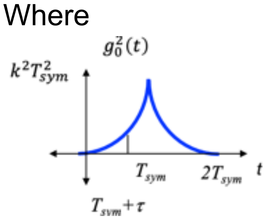
- Use matched filter to find **time τ of peak symbol power**
- 



- Develop **adaptive method** to update τ in nth symbol period
- Maximize objective function J = ½ $y^2(t)$

$$y(t) = \underbrace{g(t) * h(t)}_{g_0(t)} + \underbrace{w(t) * h(t)}_{n(t)}$$

Where



and n(t) is gaussian random signal with mean 0 and var σ^2/T_{sym}

$$\tau[n+1] = \tau[n] + \mu \frac{d}{d\tau} J\big(y(nT_{sym} + \tau) \big)\Big]_{\tau=\tau[n]}$$

$$\tau[n+1] = \tau[n] + \mu\, y(nT_{sym} + \tau[n]) \underset{?}{\frac{d}{dt} y(t)}\Big]_{t=nT_{sym}+\tau[n]}$$